# Low-Complexity Iterative Channel Estimation for Turbo Receivers

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***Abstract*—This letter discusses a receiver-side channel estimation algorithm well-suited to turbo equalizers for multiple-input multiple-output (MIMO) systems. The proposed technique is a Kalman-based channel estimator that runs on parallel single-input single-output (SISO) channels. Soft-decision-feedback interference cancellation is utilized to reduce the MIMO channel estimation problem into multiple SISO channel estimation problems. Unlike existing methods, however, the inherent correlation that exists among the output samples of the successive interference canceller is suppressed via careful puncturing of observation samples. The quality of soft decisions and channel estimates are also continuously monitored and incorporated in the Kalman filter update process.**

***Index Terms*—MIMO-OFDM, spatial multiplexing, channel estimation, soft information.**

## I. Introduction

We consider multiple input multiple output (MIMO) antenna techniques. A MIMO receiver executes equalization of received signals that have been transmitted in parallel through multiple channel links. However, imperfect channel state information (CSI) degrades the MIMO receiver performance significantly, easily wiping out the performance gain obtained via elaborate coding and signal processing schemes. Thus, accurate channel estimation is critical in realizing the full performance potential of MIMO systems. The detrimental impact of imperfect CSI on MIMO reception is well known and is continuing to be a major issue in achieving reliable wireless communication [1], [2].

To address this problem, iterative channel estimation techniques have been considered that jointly work with turbo equalization [3] -[15]. Turbo equalization, also known as iterative detection and decoding (IDD), is effective in improving receiver performance in the presence of interference [7], [8]. Broadly speaking, two kinds of iterative channel estimation algorithms have been discussed in the literature: expectation-maximization (EM) based maximum likelihood (ML) estimation [9]–[11] and Kalman-based minimum-mean-square-error (MMSE) estimation [12]–[15]. Although these algorithms typically provide solid performance, high computational complexity and iteration latency usually become a major issue when attempting practical implementation. The implementation challenge becomes greater as the number of wireless antenna links increases in the MIMO system. A major source of the computational load is a matrix inversion operation that plagues both EM-based approaches and Kalman-based methods.

In this letter, we focus on low-complexity design based on a Kalman-based channel estimation algorithm. To avoid the matrix inversion, the proposed estimator relies on soft-feedback-decision-based cancelation of interferences from adjacent antennas, just as in [16]. Our focus in this paper, however, is channel estimation. Our algorithm continuously monitors the level of potential residual interference by assessing the quality of the soft decisions and channel estimates utilized in the feedback cancelation; if the utilized soft decision is deemed unreliable, then an appropriate penalty is applied to it by temporarily raising the variance of the noise-plus-interference signal in the Kalman-based channel estimation process. In addition, to address the inherent correlation that exits between successive Kalman-estimator inputs, we adopt the refined innovation method of [14], [15] via irregular puncturing of the channel estimator input sequence. In this sense, the method proposed in this letter can be viewed as a low-complexity version of our previous channel estimation scheme [14], [15]. However, the method of this letter does not assume the specific pipeline architecture of [14]. In demonstrating the viability of the proposed scheme, we consider a MIMO method used in conjunction with orthogonal frequency division multiplexing (OFDM) complying with the IEEE 802.11n WLAN standard [17]. Frequency-selective channels are assumed since they pose the biggest challenges in reliable channel estimation.

Section II discusses the system model and briefly touches upon the IDD method used in simulating the MIMO-OFDM system performance. Section III discusses the proposed low-complexity soft-decision-directed channel estimation method. Section IV presents computation complexity evaluation and packet error rate (PER) simulation results for performance comparison, and finally conclusions are drawn in Section V.

## II. System Model

For practical algorithm design, the IEEE 802.11n wireless LAN system is assumed which supports spatial-multiplex (SM) MIMO-OFDM techniques [17]. In a MIMO-OFDM transmitter (TX) complying with the IEEE 802.11n standard, a convolutional encoder output bit sequence gets divided into $N_t$ spatial streams. Each spatial stream is interleaved separately, and the interleaved bit stream is converted into a symbol stream via M-ary quadrature amplitude modulation (M-QAM). The M-QAM symbol sequence in each spatial stream is sent through a wireless channel by an OFDM TX. The received

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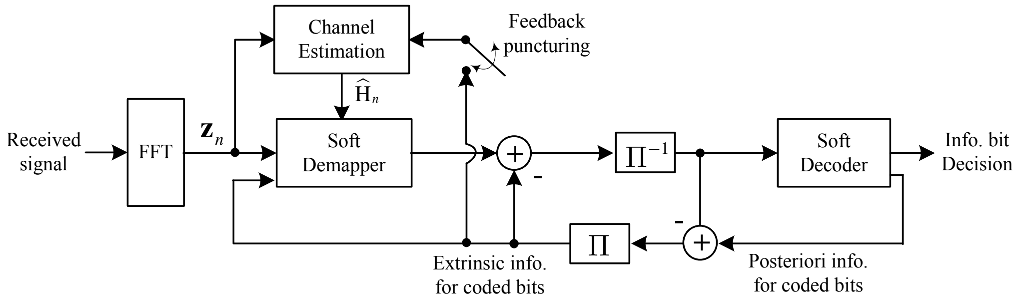


Fig. 1. Block diagram of the turbo receiver and the soft-decision-directed channel estimator.

signal on a particular frequency tone can be written as

$$\mathbf{z}_n = \mathbf{H}_n \mathbf{s}_n + \mathbf{n}_n, \tag{1}$$

where $\mathbf{z_n} = [z_n^{(1)}, z_n^{(2)}, \cdots, z_n^{(N_r)}]^T$ denotes the signal samples received at the $N_r$ receiver (RX) antennas, $\mathbf{H}_n$ is the channel response matrix corresponding to all TX-RX antenna links with its $(r,t)$ element representing the link between the $t^{th}$ TX antenna and the $r^{th}$ RX antenna, $\mathbf{s_n} = [s_n^{(1)}, s_n^{(2)}, \cdots, s_n^{(N_t)}]^T$ are the transmitted symbols from the $N_t$ TX antennas, and $\mathbf{n}_n$ is an $N_r \times 1$ vector of additive white Gaussian noise (AWGN) samples with zero mean and variance $\mathcal{N}_0$. An IEEE 802.11n packet is also provided with a special training sequence for initial channel estimation [17]. The initial channel estimation is performed by the least square (LS) estimator as in $\hat{\mathbf{H}}_{\mathbf{init}} = \mathbf{Z_{tr}} \mathbf{S}_{tr}^T \left(\mathbf{S}_{tr}\, \mathbf{S}_{tr}^T\right)^{-1}$, where $\mathbf{S}_{tr}$ and $\mathbf{Z}_{tr}$ are the training symbol matrix and the training received signal matrix, respectively. The symbols $\mathbf{S}_{tr}$ is a predetermined orthogonal-matrix with which direct matrix inversion is avoided.

The turbo-equalization strategy can be applied to the MIMO-OFDM communication receiver [7], [8]. The turbo equalizer consists of a soft-input soft-output (SISO) decoder and a SISO demapper as well as a channel estimator, as shown in Fig. 1. In our simulator, the soft MMSE-demapping algorithm is used for the SISO demapper [8], and the soft-output Viterbi algorithm (SOVA) is used for the SISO decoder implementation [18]. The extrinsic log-likelihood-ratio (LLR) fed back from the decoder is utilized for channel estimation. In order to suppress correlation in the successive estimator input samples, the extrinsic LLRs are selectively fed to the estimator by a switching scheme, as will be elaborated in the next section.

## III. Low-Complexity Kalman-Based Channel Estimation

### A. *Linear Successive Interference Cancellation Based on Soft Decisions*

The transmitted signal through a given TX-RX antenna link is affected by interferences coming from other TX antennas. One way to remove interference is to linearly cancel it out using soft-decision feedback and the most recent channel estimate [16]. The iterative detection algorithm with successive interference cancellation (SIC) in [16] generates soft decisions while including the effect of soft-decision errors within the noise variance estimate. Monitoring the potential decision errors is effective in mitigating error propagation observed in decision-feedback schemes. The channel estimation error and the feedback decision error affect each other and cause error propagation. Our previous work in [14] also discussed potential interference from the channel estimation error and the soft-decision error and a strategy to handle it in optimal decision-feedback estimator design. In this work, we focus on a low-complexity technique that avoids matrix inversion during channel estimation. The proposed channel estimator utilizes SIC to resolve complex MIMO estimation into a single-input single-output setup. While applying SIC, the potential impact of the channel estimation and feedback soft-decision errors is monitored by estimating and tracking the interference variance.

The MIMO channel interference is cancelled using the soft decisions fed back from the soft decoder. The soft symbol decision $\tilde{s}$ is defined as the average of the constellation symbols with the averaging based on the estimated symbol probabilities, i.e., $\tilde{s} = \sum_{s_j \in \mathcal{A}} s_j P(s_j)$ where $P(s_j)$ can be taken as the "extrinsic probability" obtained from conversion of the available extrinsic LLR.

We notice that not all symbols in the M-QAM constellation has significant probabilities. Accordingly, only those QAM symbols associated with high extrinsic probabilities are considered. For example, the soft symbol decision is approximated as $\tilde{s} = \sum_{s_j \in A^{\{4\}}} s_j P_4(s_j)$ where $P_4(s_j)$ is defined as the normalized probability $P_4(s_j) = P(s_j) / \left( \sum_{s_j \in A^{\{4\}}} P(s_j) \right)$, and $A^{\{4\}}$ indicates an M-QAM sub-constellation containing only four highly probable symbols. This approximation leads to a reduced computational load with little performance loss. The SIC scheme using soft decision can be summarized as

$$\begin{aligned} \tilde{z}_n^{(r,t)}[i] &= z_n^{(r)} - \sum_{\substack{j=1 \\ j \neq t}}^{N_t} \hat{h}_n^{(j,r)}[i-1] \tilde{s}_n^{(j,r)}[i] \\ &= h^{(r,t)} \tilde{s}_n^{(r,t)}[i] + u_n^{(r,t)}[i] \end{aligned} \tag{2}$$

where $\tilde{z}^{(r,t)}$ denotes the received signal associated with the $t^{th}$ TX– $r^{th}$ RX channel link. Subscript $n$ corresponds to the received OFDM symbol time index, and variable $i$ points to the IDD iteration number ($0 \leq i \leq N_{itr}$), where $N_{itr}$ denotes the number of the IDD iterations required to achieve satisfactory error rate performance. IDD iterations are applied sequentially to each OFDM symbol; for each $n$ value, index $i$ increases from 0 to $N_{itr}$, as illustrated in Fig. 2. The estimate $\hat{h}_n^{(r,t)}[i-1]$ is the available up-to-date channel estimate,

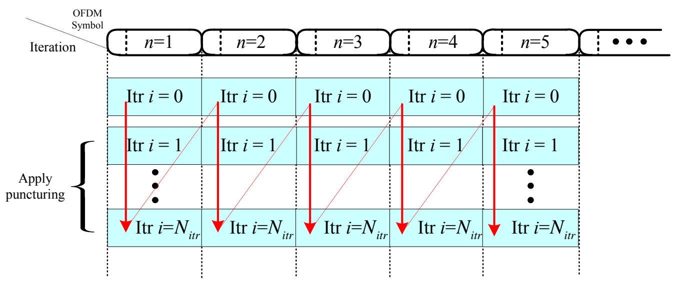


Fig. 2. OFDM-symbol processing procedure in IDD.

whether from the initial channel estimation process or from the previous channel estimation cycle, and $u_n^{(r,t)}[i]$ represents noise plus any potential remaining interference after SIC is attempted at the $i^{th}$ IDD iteration. The estimate for each TX-RX channel link is obtained assuming all parallel channel responses are independent. In developing a Kalman-based channel estimation algorithm, one would naturally try to set up an innovation sequence using the observation signal $\tilde{z}$, but as discussed below, significant caution is in order.

### *B. Soft-Decision-Directed Kalman Channel Estimator*

An innovation sequence is a white sequence that is a causal and casually invertible linear transformation of the observation sequence [19]. Unfortunately, the observation signal $\tilde{z}$ cannot lead to a reliable innovation sequence to develop a Kalman filter. Let us discuss why. In an attempt to derive a scalar version of the Kalman channel estimator, one would set up an innovation sequence as

$$\begin{aligned} x_n^{(r,t)}[i] &\triangleq \tilde{z}_n^{(r)}[i] - \hat{h}_n^{(r,t)}[i-1]\tilde{s}_n^{(t)}[i] \quad (3)\\ &= \sum_{j=1}^{N_t} \varepsilon_n^{(r,j)}[i-1]s_n^{(t)} - \sum_{j=1}^{N_t} h^{(r,j)}e_n^{(j)}[i] + n_n^{(r)} \end{aligned}$$

where $e_n[i]$ and $\varepsilon_n^{(r,j)}[i]$ are defined as $e_n[i] \triangleq s_n - \tilde{s}_n[i]$ and $\varepsilon_n^{(r,j)}[i] \triangleq h^{(r,j)} - \hat{h}_n^{(r,j)}[i]$ respectively. With an understanding that we focus on a specific channel link, the TX–RX antenna indices $t$ and $r$ are dropped from (4) to reduce notation cluttering. The sequence $[x_0[0], .., x_0[N_{itr}], x_1[0], ..., x_1[N_{itr}], ..., x_n[0], ..., x_n[N_{itr}]]$ would be an innovation sequence, if $\varepsilon_n[i]$ and $e_n[i]$ in (4) are uncorrelated over time $n$ and iteration $i$. However, it is not difficult to see $x_n[i]$ is a correlated sequence. First, the AWGN noise component cannot be decorrelated during iterations, because the noise sample $n_n$ remains fixed for $i = 1, .., N_{itr}$. Accordingly, the amount of correlation in the sequence $x_n$ is at least $\mathcal{N}_o$ since $E[x_n[i-k]x_n^*[i]] \geq \mathcal{N}_o$ $(k > 0)$. Secondly, when the IDD makes erroneous decisions, $\varepsilon_n[i]$ and $e_n[i]$ are correlated with $\varepsilon_n[i-k]$ and $e_n[i-k]$, respectively, correlated errors circulate among IDD modules as well as the channel estimator during iterations [14].

In an effort to whiten the channel estimator input sequence, an useful insight is gained from the well-known Gram-Schmidt procedure. Write $x_n^{'}[i] = x_n[i] - \frac{\langle x_n[i], x_n[i-k]\rangle}{|x_n[i-k]|^2} x_n[i-k], (k > 0)$, where $< a, b >$ denotes the inner product: $< a, b >= Re(a)Re(b) + Im(a)Im(b)$. Assuming there is no correlation between $Re(x_n)$ and $Im(x_n)$, we can write $E[x_n[i]\,x_n^*[i-k]] = E\left[x_n^{'}[i]\,x_n^*[i-k]\right] + E\left[|x_n[i-k]|^2 \frac{\langle x_n[i], x_n[i-k]\rangle}{|x_n[i-k]|^2}\right] = E[\langle x_n[i], x_n^*[i-k]\rangle]$. For the current input $x_n[i]$, the highest correlation exists with $x_n[i-1]$. So, define $\beta_n[i] \triangleq \langle x_n[i], x_n[i-1]\rangle$, which is used to monitor the amount of correlation. Now, the main idea is to filter out the samples with correlated errors, i.e., drop the samples for which $|\ \beta_n[i]\ | > c\mathcal{N}_o$. The positive constant $c$ is an adjustable parameter that controls the threshold level. In this way, the samples with high correlated errors are blocked from entering the channel estimator; this is done by defining the switching function:

$$\begin{aligned} &w_n[i] = 1, \ \ when\ i = 1\ or\ |\beta_n[i]| \leq c\mathcal{N}_o\ for\ 2 \leq i \leq N_{itr}\\ &w_n[i] = 0, \ \ otherwise. \end{aligned}$$

The Kalma filter halts the channel update process when $w_n[i] = 0$. The decision from the initial iteration $(i \neq 0)$ is not utilized in estimation as it is typically not reliable enough, whereas the decision at $i = 1$ is always used.

First, a sequential linear MMSE-based channel estimator is given as follows:

$$\begin{aligned} \hat{h}_n[i] &\triangleq \widehat{E}[h|x_n[i]] + \\ &\quad\ \widehat{E}[h|x_n[i-1], .., x_n[1], x_{n-1}[N_{itr}], ..., x_0[1]] \\ &= K_n[i]x_n[i] + \hat{h}_n[i-1] \qquad (4) \end{aligned}$$

where $K_n[i]$ is the Kalman-filter gain, which can be derived from the orthogonality principle. Start with $\overline{(h - K_n[i]x_n[i])\,x_n^*[i]} = 0$ or $\overline{hx_n^*[i]} = \overline{(K_n[i]x_n[i])\,x_n^*[i]}$. Since $\overline{hx_n^*[i]} = \overline{|\varepsilon_n[i-1]|^2\tilde{s}_n^*[i]}$ and $\overline{x_n[i]x_n^*[i]} = \overline{|\tilde{s}_n[i]\varepsilon_n[i-1]|^2} + \overline{|u_n[i]|^2}$, we can write

$$K_n[i] = \frac{M_n[i-1]\tilde{s}_n^*[i]}{M_n[i-1]\,|\tilde{s}_n[i]|^2 + \sigma_u^2[i]} \qquad (5)$$

where $M_n[i]$ is the channel estimation error variance given as $M_n[i] \triangleq \overline{|h - \widehat{h}_n[i]|^2}$, and $\sigma_u^2[i]$ is the noise-plus-interference variance of $u_n[i]$. Utilizing (4), we can express $M_n[i]$ recursively as $M_n[i] = (1 - K_n[i]\tilde{s}_n[i])M_n[i-1]$. Finally, the sequential Kalman-based estimator incorporating the puncturing scheme of (4) can be summarized as

$$\begin{aligned} K_n^{'}[i] &= \frac{w_n[i]\ M_n[i-1]}{M_n[i-1] + \sigma_u^2[i]/\,|\tilde{s}_n[i]|^2}, \qquad (6)\\ M_n[i] &= (1 - K_n^{'}[i])M_n[i-1] \qquad (7)\\ \hat{h}_n[i] &= K_n^{'}[i]\check{h}_n[i] + (1 - K_n^{'}[i])\hat{h}_n[i-1] \qquad (8) \end{aligned}$$

where $K_n^{'}[i]$ is a modified form of the Kalman-filter gain with puncturing, and $\check{h}_n[i]$ denotes the channel estimate based on the current observation. The initial estimate $\hat{h}_{-1}$ is obtained from the initial preamble-based channel estimator. Also, the initial error variance $M_{-1}$ is derived from the mean square error (MSE) analysis in [20] as $M_{-1} = |\hat{h}_{-1}|^2/\{|\hat{h}_{-1}|^2E_s/(N_t\mathcal{N}_o) + 1\}$ where $E_s$ is the signal power of the training symbol. Finally, in order to complete the estimator (6)-(8), we need to find : (i) the noise-plus-interference variance $\sigma_u^2[i]$ and (ii) the channel estimate $\check{h}_n[i]$.

The soft feedback decisions have different levels of reliability, and this needs be taken into account. From (2), the

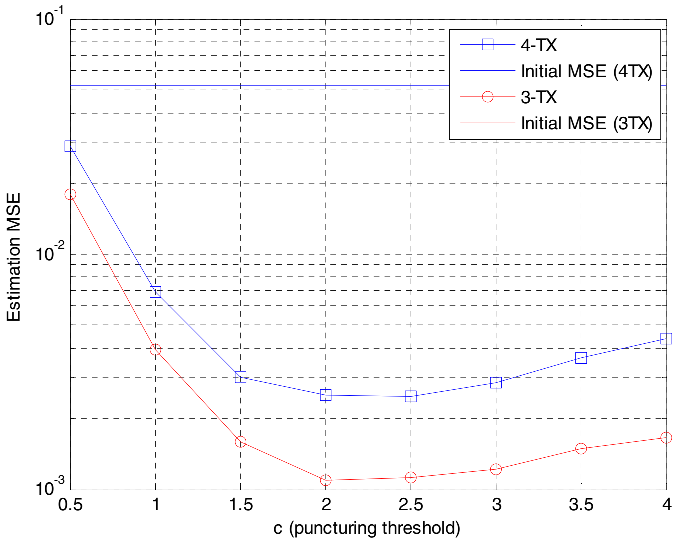


Fig. 3. Threshold parameter $c$ optimization.

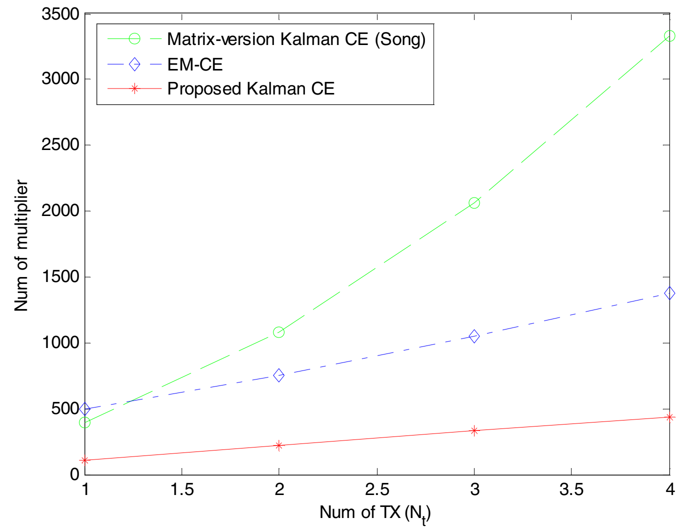


Fig. 4. Computation complexity comparison: number of multipliers to generate $N_t \times N_r$ channel estimates ($N_r = 4, N_b = 4$).

variance of the noise-plus-interference can be written as

$$\sigma_u^2[i] = \mathcal{N}_o + \sum_{\substack{j=1 \\ i \neq t}}^{N_t} \Big( |h^{(j)}|^2 \overline{|s_n^{(j)} - \tilde{s}_n^{(j)}[i]|^2} + \overline{|h^{(j)} - \hat{h}_n^{(j)}[i-1]|^2}\ |\tilde{s}_n^{(j)}[i]|^2 \Big) \quad (9)$$

$$\approx \mathcal{N}_o + \sum_{\substack{j=1 \\ j \neq t}}^{N_t} \Big( M_n^{(j)}[i-1] + \overline{|\hat{h}_n^{(j)}[i-1]|^2} \Big) \cdot \overline{|s_n^{(j)} - \tilde{s}_n^{(j)}[i]|^2} + M_n^{(j)}[i-1]\ |\tilde{s}_n^{(j)}[i]|^2 \quad (10)$$

Note $\overline{|h^{(j)} - \hat{h}_n^{(j)}[i-1]|^2}$ can be provided by the Kalman filter parameter $M_n[i]$, which indicates the quality of the estimate. The soft decision variance is evaluated as $\overline{|s - \tilde{s}|^2} = \sum_{s_j \in A\{4\}} |s_j - \tilde{s}|^2 P_4(s_j)$, and an approximation $\overline{|h|^2} \approx |\hat{h}_n[i-1] + \varepsilon_n[i-1]|^2$ is used in solving (10).

Next, using the extrinsic probabilities of M-QAM symbols, $\check{h}_n[i]$ can be found as

$$\check{h}_n[i] = \sum_{s_j \in \mathcal{A}\{4\}} \frac{s_j^* \tilde{z}_n[i]}{|s_j|^2 + \sigma_u^2[i]/|\hat{h}_n[i-1]|^2} P_4(s_j). \quad (11)$$

Note that (11) is the linear MMSE estimator derived for a fixed QAM symbol $s$, averaged according to $P(s)$. This can be considered as an optimal estimator given $\tilde{z}_n[i]$ and the extrinsic probabilities. Another possible variation for the estimator is

$$\check{h}_n[i] = \sum_{s_j \in \mathcal{A}\{4\}} \frac{\tilde{z}_n[i]}{s_j} P_4(s_j)\ , \quad (12)$$

which is basically a weighted sum of the zero-forcing estimators. In the sequel, the estimator (12) is used in executing the recursive estimator of (6)-(8) and generating simulation results, as it was seen to be equally effective during our investigation. We notice that the noise-interference variance $\sigma_u^2$ plays a key role in the Kalman update process. Based on the soft-decision reliability, the Kalman-filter solution is adaptively controlled by $\sigma_u^2$. Also, note that the constant $c$ controls the estimation convergence speed and the amount of correlated errors, which can be traded in our channel estimator design. The existence of the optimum value for $c$ is demonstrated with initial channel estimation error simulation as shown in Fig. 3. Our MSE simulation results in $3 \times 3$ and $4 \times 4$ systems show that the optimal $c$ value is at 2 and 2.5, respectively.

## IV. Performance Evaluation

The Kalman-based channel estimator [12] and the EM channel estimator [9] are compared to the proposed low-complexity channel estimator in terms of computation complexity and PER performance.

### A. Complexity Comparison with Existing Channel Estimators

For performance comparison, the EM-based decision-directed (DD) estimator introduced in [9] as a variant of the EM estimator is set up as $\check{\mathbf{h}}_n^{(r)} = (\widetilde{\mathbf{S}}_n^H \widetilde{\mathbf{S}}_n)^{-1} \widetilde{\mathbf{S}}_n^H \mathbf{z}_n^{(r)}$, where $\mathbf{z}_n^{(r)}$ is an $N_b \times 1$ received signal vector buffering $N_b$ OFDM symbols at the $r^{th}$ RX, $\widetilde{\mathbf{S}}_n$ is a $N_b \times N_t$ matrix consisting of decoder-feedback soft symbols, $\mathbf{h}^{(r)}$ is a $N_t \times 1$ channel vector. This estimate is blended with the training-based channel estimate by the combining method of [10].

The iterative Kalman estimator using soft-decision feedback has been introduced in [12]. The proposed algorithm is compared with two versions of Kalman-based algorithms: a conventional scalar Kalman estimator and an optimum Kalman MIMO (matrix/vector version) channel estimator. The same SIC step is applied to the conventional scalar Kalman estimator, but the estimator only assumes additive random noise with variance $\mathcal{N}_o$, not considering the residual interference power. So, in the compared conventional scalar Kalman scheme, the gain update equation of (6) is replaced with $K_n^{'}[i] = \frac{M_n[i-1]}{M_n[i-1] + \mathcal{N}_o/|\tilde{s}_n[i]|^2}$, while (7) and (8) remain the same. Also, the mean-symbol-based zero-forcing method [4] with $\check{h}_n[i] = \frac{\tilde{z}_n[i]}{\tilde{s}_n[i]}$ is employed instead of (12). As for the comparison with the optimum

MIMO Kalman-based estimator with full matrix operations, the Kalman estimator of [12] is modified. The scheme of [12] is designed to handle soft decisions with varying qualities by adjusting the effective noise variance but remains unaware of the effect of correlation among successive estimator input samples caused by channel estimation errors. Also, since the scheme of [12] was introduced in the context of intersymbol interference (ISI), we modify it here to handle the multi-input single-output (MISO) channel by setting up (i) the noise-interference variance update matrix $\mathbf{Q}_n^{(r)} = \mathrm{diag}[\sum_{t=1}^{N_t} (p_n(t,t) + |\hat{h}_t[n-1]|^2)\sigma_s^2(n,i,t)]$, $i = 1,..,N_b$, (ii) the MMSE estimator $\mathbf{A}_n^{(r)} = (\widetilde{\mathbf{S}}_n^H(\mathbf{Q}_n^{(r)}+\mathcal{N}_o\mathbf{I}_{N_b})^{-1}\widetilde{\mathbf{S}}_n+\mathbf{P}_{n-1}^{(r)-1})^{-1}\widetilde{\mathbf{S}}_n^H(\mathbf{Q}_n^{(r)}+\mathcal{N}_o\mathbf{I}_{N_b})^{-1}$, (iii) the estimation error covariance matrix $\mathbf{P}_n^{(r)} = (\mathbf{I}_{N_t} - \mathbf{A}_n^{(r)}\widetilde{\mathbf{S}}_n)\mathbf{P}_{n-1}^{(r)}$, and (vi) the Kalman sequential update equation $\hat{\mathbf{h}}_n^{(r)} = \mathbf{A}_n^{(r)}\mathbf{z}_n^{(r)} + (\mathbf{I} - \mathbf{A}_n^{(r)}\widetilde{\mathbf{S}}_n)\hat{\mathbf{h}}_{n-1}$. To extend the matrix-version Kalman MISO estimator to the case of MIMO channel estimation, we repeat the four steps above for each RX antenna signal vector.

Existing channel estimators for MIMO channel links are designed based on matrix operations, and the computation load is largely caused by the corresponding multiplication operations. Assuming $N_r = 4$ and $N_b = 4$ (buffer size), the total number of real multipliers versus the number of TXs is presented in Fig. 4. The proposed algorithm requires $23N_tN_r + 18N_t$ multipliers but no matrix inversions. Assume the number of real multipliers for a matrix inversion goes as $\mathbf{O}(N_t^3)$ (as in the Gauss-Jordan elimination method). The EM estimator uses a total of $\mathbf{O}(N_t^3)+4N_t^2N_b+4N_t(3N_bN_r+N_r)+N_bN_r+4N_b^3$ multipliers, and the Kalman estimator utilizes $2N_r\mathbf{O}(N_t^3) + 4N_t^2(N_r + 2N_bN_r) + N_t(10N_bN_r + N_b^2N_r + 4N_r + 3N_b)$ multipliers. Overall, the matrix inversion causes a major computation load proportional to $N_t^3$. Fig. 4 shows that with $N_t = 4$, the proposed algorithm can operate with $1/7$ and $1/3$ of the computation load of the optimal Kalman estimator and of the EM estimator, respectively.

### B. PER Simulation Results

In this section, we compare the receiver performance using the different channel estimation methods described in Section IV-A. The performance comparison is made in terms of PER versus SNR curves. Performance is evaluated for $3 \times 3$ and $4 \times 4$ 16-QAM SM-MIMO-OFDM systems. A rate 1/2 convolutional code is used complying with the IEEE 802.11n specification [17]. The transmitter sends packets that contain 1000 bytes of information. Required SNRs are compared with at the 1% PER level, at which practical WLAN systems reasonably operate. The simulated MIMO multi-path channel was modelled as a quasi-static multi-path channel with an exponential power profile with a root-mean-square (rms) delay of 50 ns. The channel responses are assumed uncorrelated across different antenna links.

For the sake of comparison, we also present PER curves for the cases of perfect CSI and perfect decision (PD) feedback knowledge, respectively. In Fig. 5, the $3 \times 3$ SM-MIMO-OFDM system using only initial channel estimation (labelled 'initial CE') shows a 3 dB SNR degradation at a $10^{-2}$ PER

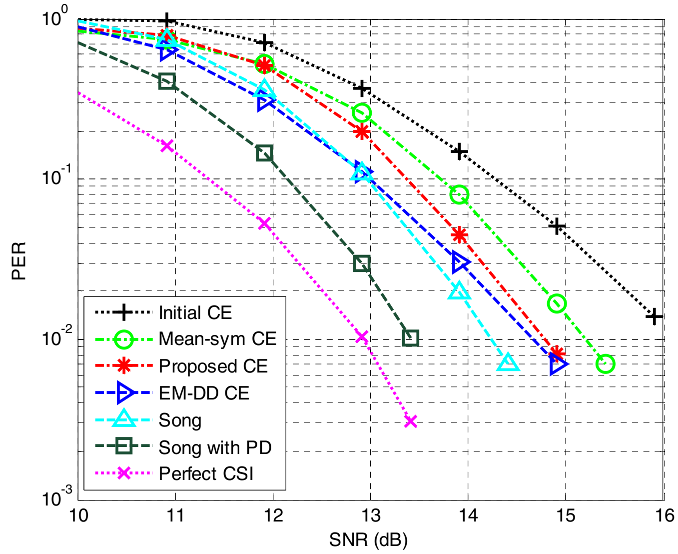


Fig. 5. PER simulations in the $3 \times 3$ SM-MIMO-OFDM turbo receiver (7 iterations, $c = 2$).

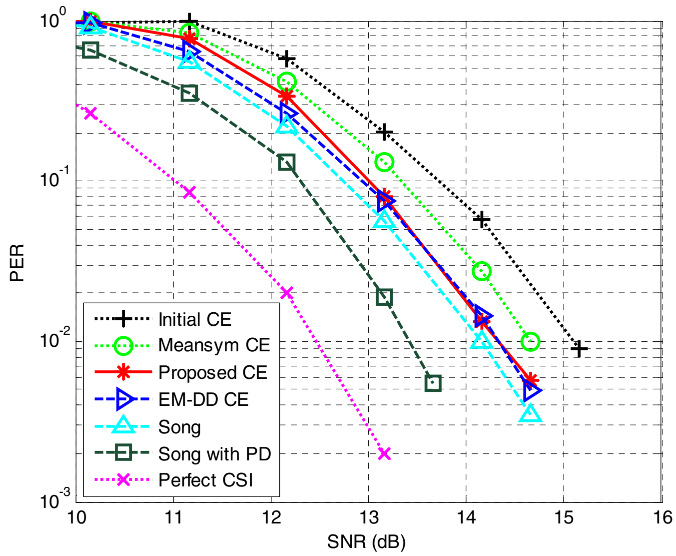


Fig. 6. PER simulations in the $4 \times 4$ SM-MIMO-OFDM turbo receiver (9 iterations, $c = 2.5$).

relative to the perfect CSI curve. Considering computational complexity, the proposed algorithm shows competitive performance. The proposed estimator recovers a 1.2 dB of the 3 dB loss incurred by the initial estimation only. As compared to algorithms using full-matrix operation, the "Song" and EM-DD methods, the proposed algorithm lags behind the former by 0.5 dB but performs almost comparably to the latter while the computational load is much less. Fig. 6, corresponding to the $4 \times 4$ SM-MIMO-OFDM system, shows that the proposed algorithm suffers only a slight performance degradation with respect to the Song method (Kalman MIMO estimator) while providing essentially the same performance as the EM-DD MIMO estimator. Considering the much higher levels of complexity required for implementing the Song and the EM-DD methods, it is safe to say that the proposed algorithm enjoys favorable complexity/performance tradeoff. Meanwhile, the mean-symbol-based algorithm (labelled 'Mean-symbol CE') lags behind noticeably in its performance relative to the proposed algorithm.

## V. Conclusions

A low-complexity Kalman-based channel estimation algorithm geared to turbo equalizers for MIMO systems has been proposed. As shown in the performance evaluation results, MIMO receivers suffer significant performance loss due to imperfect CSI. It is desired to recover this loss, but existing MIMO channel estimation algorithms are computationally intensive. The proposed low-complexity algorithm resolves the MIMO channel estimation problem into multiple SISO channel estimation problems using SIC. The proposed channel estimator then tracks the combined power of noise and residual interference due to potential miss-cancellation in an effort to minimize the effect of error propagation. The scheme also utilizes a puncturing technique that drops correlated observation samples at the channel estimator input. Kalman-based channel estimators typically operate on the premise that the estimator input is an innovation sequence, a fallacy whenever channel estimation errors exist. Thus, puncturing has a tendency to correct this erroneous and harmful assumption. The proposed algorithm is well suited to iterative receivers, providing robust performance with a low computation load. The robust performance has been demonstrated via PER comparison with existing Kalman-based estimators as well as a representative EM-based estimator.

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